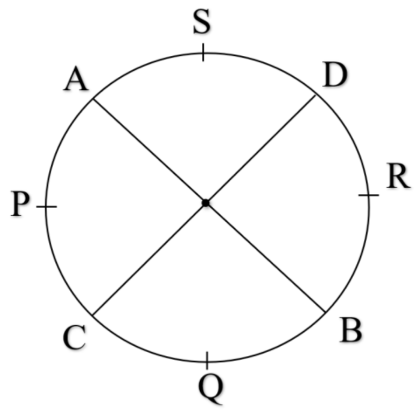
17. Circle: Chord and Arc

1. ‘O’ is the centre of the circle whose diameters are seg AB and seg CD. If $m\angle AOC = 65^0$ then find the measures of following arc.



(i) Arc CQB **(ii) Arc ASD**

(iii) Arc CQB **(iv) Arc APC**

(v) Arc ADC **(vi) Arc ACD**

Solution : $m\angle AOC = 65^0$

(i) Measure of arc BRD = m (arc BRD) = $m\angle BOD$

…… (Corresponding central angle)

$= m\angle AOC$ …… (Opposite angle)

$= 65^0$

$\therefore$ The measure of arc BRD is 65^0.

(ii) Measure of arc ASD = m (arc ASD)

$= m\angle AOD$

$= 180^0 - m\angle AOC$

…... (Angles in a linear pair)

$= 180^0 - 65^0$

$= 115^0$

$\therefore$ **The measure of arc ASD is 115^0.**

(iii) Measure of arc CQB = m (arc CQB)

$= m \angle COB$

$= 180^0 - m \angle AOC$

…. (Angles in a linear pair)

$= 180^0 - 65^0$

$= 115^0$

(iv) Measure of are APC = m (arc APC)

$= m \angle AOC$

$= 65^0$ **…… (Given)**

(v) Measure of arc ADC = m (arc ADC)

$= 360^0 - m(arc\ APC)$

$= 360^0 - m \angle AOC$

$= 360^0 - 65^0$

$= 295^0$

(vi) Measure of arc ACD = m (arc ACD)

$= 360^0 - m\ (arc\ ASD)$

$= 360^0 - m\angle AOD$

$= 360^0 - (180 - m\angle AOC)$

But, $m\angle AOD = 180 - m\angle AOC$

..... (Angles in a linear pair)

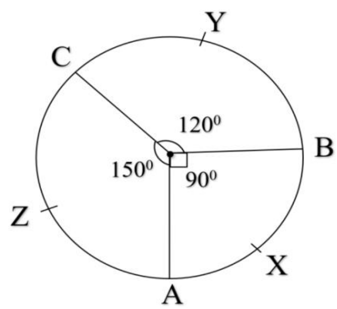
$= 360^0 - 180 + m\angle AOC$

$= 180^0 + m\angle AOC$

$= 180^0 + 65^0$

$= 245^0$

2. In the adjacent figure, 'O' is the centre of the circle whose radii are seg OB and seg OC. If seg OB $\perp$ seg OA and $m\angle BOC = 120^0$ then find the measures of following arc.



(i) m (arc AXB) (ii) m (arc BYC)

(iii) m (arc CZA) (iv) m (arc ABC)

(v) m (arc BAC) (vi) m (arc BCA)

Solution : Given :

$m\angle BOC = 120^0, m\angle AOC = 150^0, m\angle AOB = 90^0$

(i) $m\,(arc\ AXB) = m\angle AOB = 90^0$ ..... (Given)

(ii) $m\,(arc\ BYC) = m\angle BOC = 120^0$ ..... (Given)

(iii) $m\ (\text{arc CZA}) = m\angle AOC = 150^0$ ...... (Given)

(iv) $m\ (\text{arc ABC}) = 360^0 - m(\text{arc CZA})$

$= 360^0 - m\angle AOC$

$= 360^0 - 150^0$

$= 210^0$

(v) $m\ (\text{arc BAC}) = 360^0 - m(\text{arc BYC})$

$= 360^0 - m\angle BOC$

$= 360^0 - 120^0$

$= 240^0$

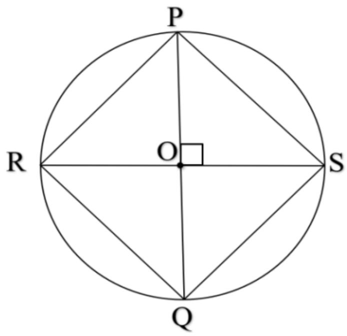
(vi) $m\ (\text{arc BAC}) = 360^0 - m(\text{arc AXB})$

$= 360^0 - m\angle AOB$

$= 360^0 - 90^0$

$= 270^0$

3. In a circle with centre 'O' seg PQ and seg RS are perpendicular to each other. Then show that



(i) Chord PS $\cong$ Chord SQ

(ii) Chord PR $\cong$ Chord RQ

Solution : In a circle with centre ‘O’ seg PQ and seg RS are perpendicular to each other.

∴ Seg PQ $\perp$ seg RS

∴ $m\angle POS = m\angle SOQ = m\angle QOR = m\angle ROP = 90^0$

$m\angle POS = m\angle SOQ = 90^0$

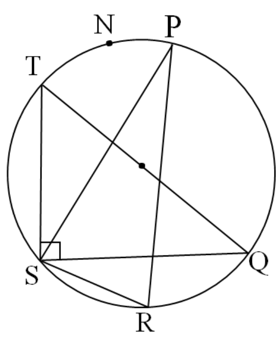
∴ m (arc PS) = m (arc SQ)

…… (Arcs are same measures)

∴ arc PS $\cong$ arc SQ

∴ Chord PS $\cong$ Chord SQ ….. (The chords corresponding to congruent arcs are congruent.)

4. Observe the given figure and give the answers of the following questions



(i) Write the name of right angled triangle.

(ii) What is the name of angle corresponding to the arc SRQ.

(iii) Which is the angle made by arc SNP ?

(iv) Which is the angle made by arc TPQ ?

(v) Which is the angle made by arc SR ?

(vi) Which is the angle corresponding to the arc PQ ?

Solution : (i) In the figure, Δ TSQ is the right angled triangle because m $\angle$ TSQ $= 90^0$.

(ii) $\angle$ STQ is the corresponding angle to the arc SRQ.

(iii) $\angle$ PRS is the angle made by arc SNP.

(iv) Arc SNP is made by the $\angle$ TSQ.

(v) Arc SR is made by the $\angle$ SPR.

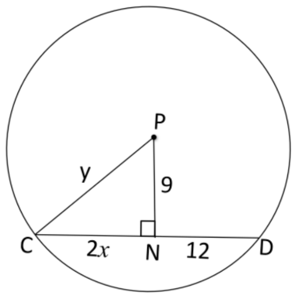
(vi) $\angle$ PSQ is the corresponding angle to the arc PQ.

5. In the figure, P is the centre of the circle. N is the midpoint of chord CD.

l (CN) $= 2x$, l (ND) $= 12$

l (PN) $= 9$ and l (PC) $=$ y then

find the value of x and y



Solution : P is the centre of the circle. Seg CD is the chord and seg PN $\perp$ chord CD

$\therefore$ The point N is the midpoint of chord CD.

….. (Property of the chord of a circle)

$\therefore$ l (CN) $=$ l (ND)

But, l (CN) $= 2x$, l (ND) $= 12$ ….. (Given)

$\therefore$ $2x = 12$

$\therefore \frac{2x}{2} = \frac{12}{2}$ ….. (Dividing both the sides by 2)

$\therefore x = 6$

$\therefore \text{CN} = 2x = 2 \times 6 = 12$

In right angled Δ PNC,

By Pythagoras theorem.

$(\text{CP})^2 = (\text{PN})^2 + (\text{CN})^2$

$\therefore \ \text{y}^2 = (9)^2 + (12)^2$

$\therefore \ \text{y}^2 = 81 + 144$

$\therefore \ \text{y}^2 = 225$

$\therefore \ \text{y}^2 = (15)^2$

$\therefore \ \text{y} = 15$

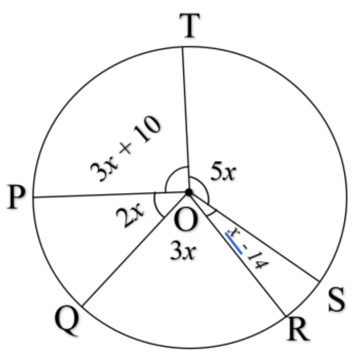
$\therefore \ x = 12$ and $\text{y} = 15$

6. In the figure alongside, 'O' is the centre of the circle.

$\text{m}\angle\text{POQ} = 2x, \text{m}\angle\text{QOR} = 3x,$

$\text{m}\angle\text{ROS} = x - 14, \text{m}\angle\text{TOS} = 5x,$

$\text{m}\angle\text{POT} = 3x + 10$ then find the value of x and the measures of all angles.



Solution : Measure of a circle is 360^0.

$\text{m}\angle\text{POQ} = 2x, \text{m}\angle\text{QOR} = 3x,$

$\mathbf{m\angle ROS = \boldsymbol{x} - 14, m\angle TOS = 5\boldsymbol{x},}$

$\mathbf{m\angle POT = 3\boldsymbol{x} + 10}$

$\therefore\ \mathbf{m\angle POQ + m\angle QOR + m\angle ROS + m\angle TOS + m\angle POT = 360^0}$

$\therefore \mathbf{2\boldsymbol{x} + 3\boldsymbol{x} + (\boldsymbol{x} - 14) + 5\boldsymbol{x} + (3\boldsymbol{x} + 10) = 360^0}$

$\therefore \mathbf{14\boldsymbol{x} - 14 + 10 = 360}$

$\therefore \mathbf{14\boldsymbol{x} - 4 = 360}$

$\therefore \mathbf{14\boldsymbol{x} = 360 + 4}$

$\therefore \mathbf{14\boldsymbol{x} = 364}$

$\therefore \boldsymbol{x} = \mathbf{\frac{364}{14}}$

$\therefore \boldsymbol{x} = \mathbf{26}$

$\therefore\ \mathbf{m\angle POQ = 2\boldsymbol{x} = 2 \times 26 = 52^0}$

$\mathbf{m\angle QOR = 3\boldsymbol{x} = 3 \times 26 = 78^0}$

$\mathbf{m\angle ROS = \boldsymbol{x} - 14 = 26 - 14 = 12^0}$

$\mathbf{m\angle TOS = 5\boldsymbol{x} = 5 \times 26 = 130^0}$

$\mathbf{m\angle POT = 3\boldsymbol{x} + 10 = 3 \times 26 + 10 = 88^0}$

7. In the adjoining figure P is the centre of the circle.

$\mathbf{m\angle MPL = 3x - 11, m\angle LPN = 4x, m\angle NPO = 4x - 7}$

then find the measures of the following arcs.

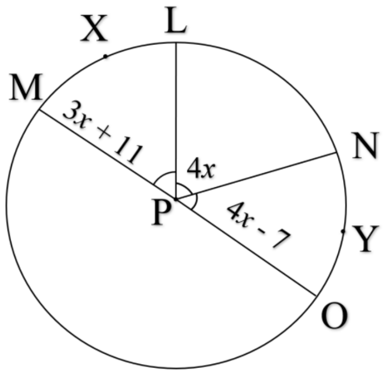
(i) m (arc MXL)

(ii) m (arc NYO)

(iii) m (arc MON)

(iv) m (arc LN)

(v) m (arc MLN)



Solution : P is the centre of the circle. Seg MO is the diameter.

$\mathbf{\angle MPO}$ **is the semicircle of the circle.**

Measure of a semicircular arc is $\mathbf{180^0}$.

$\therefore \mathbf{m\angle MPO = 180^0}$

But, $\mathbf{m\angle MPL + m\angle LPN + m\angle NPO = m\angle MPO}$

$\therefore \mathbf{(3x - 11) + 4x + (4x - 7) = 180^0}$

$$\therefore \mathbf{11x - 18 = 180}$$

$$\therefore \mathbf{11x = 180 + 18}$$

$$\therefore \mathbf{11x = 198}$$

$$\therefore \frac{\mathbf{11x}}{\mathbf{11}} = \frac{\mathbf{198}}{\mathbf{11}}$$

….. (Dividing both the sides by 11)

$$\therefore x = 18$$

$\therefore$ m $\angle$ MPL $= 3x - 11 = (3 \times 18) - 11$

$$= 54 - 11 = 43^0$$

m $\angle$ LPN $= 4x = 4 \times 18 = 72^0$

m $\angle$ NPO $= 4x - 7 = (4 \times 18) - 7 = 72 - 7 = 65^0$

(i) m (arc MXL) = m $\angle$ MPL $= 43^0$

(ii) m (arc NYO) = m $\angle$ NPO $= 65^0$

(iii) m (arc MON) = m $\angle$ MPO $+$ m $\angle$ NPO

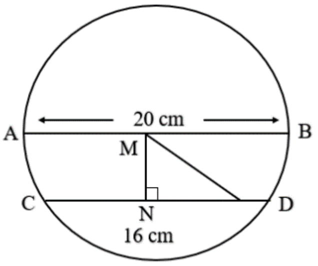
$$= 180^0 + 65^0 = 245^0$$

(iv) m (arc LN) = m $\angle$ LPN $= 72^0$

(v) m (arc MLN) = m $\angle$ MPL $+$ m $\angle$ LPN

$$= 43^0 + 72^0 = 115^0$$

8. In the figure, M is the centre of the circle MN ⊥ CD chord CD = 16 cm, diameter AB = 20 cm the find *l* (MN)



Solution :

l (CD) = 16 cm, *l* (AB) = 20 cm.

In the figure, M is the centre of the circle. AB is the diameter of the circle.

$l\ (\text{AB}) = 20 \text{ cm}$

$\therefore$ Radius $= \frac{20}{2} = 10 \text{ cm}$

$\therefore l\ (\text{MB}) = l\ (\text{MD}) = 10 \text{ cm}$

Also, Seg MN $\perp$ Chord CD

The perpendicular drawn from the centre of a circle to its chord bisects the chord.

$$\therefore l\ (\text{ND}) = \frac{1}{2} \times l\ (\text{CD})$$

$$\therefore l\ (\text{ND}) = \frac{1}{2} \times 16$$

$$l\ (\text{ND}) = 8 \text{ cm}$$

Δ MND is a right angled triangle.

$\therefore$ In Δ MND ,

By Pythagoras theorem,

$$l\ (\text{MD})^2 = l\ (\text{MN})^2 + l\ (\text{ND})^2$$

$$\therefore (10)^2 = l\ (\text{MN})^2 + (8)^2$$

$$\therefore l\ (\text{MN})^2 = (10)^2 - (8)^2$$

$$l\,(\mathrm{MN})^2 = 100 - 64$$

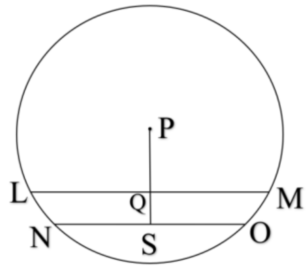
$$\therefore l\,(\mathrm{MN})^2 = 36$$

$$\therefore l\,(\mathrm{MN})^2 = (6)^2$$

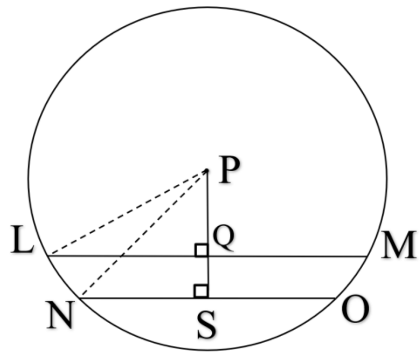
$$\therefore l\,(\mathrm{MN}) = 6$$

$\therefore$ l (MN) is 6 cm.

9. In the figure alongside, Seg PQ ⊥ chord LM, Seg PS ⊥ chord NO. *l* (CM) = 16 cm, *l* (NO) = 12 cm and the radius of a circle is 10 cm. Find *l* (QS)



Solution : Join seg PL and seg PN. *l* (LM) = 16 cm, *l* (NO) = 12 cm and the radius of a circle is 10 cm.



Seg PQ ⊥ chord LM. The perpendicular drawn from the centre of a circle to its chord bisects the chord.

$$\therefore l\,(\mathrm{LQ}) = \frac{1}{2} l\,(\mathrm{LM})$$

$$\therefore l\,(\mathrm{LQ}) = \frac{1}{2} \times 16$$

$\therefore l\,(\mathrm{LQ}) = 8\ \mathrm{cm}$

Radius of a circle = *l* (PL) = 10 cm

Δ PQL is the right angled triangle.

∴ In Δ PQL

By Pythagoras theorem,

$l\,(\mathrm{PL})^2 = l\,(\mathrm{LQ})^2 = l\,(\mathrm{PQ})^2$

$\therefore (10)^2 = (8)^2 + l\,(\mathrm{PQ})^2$

$\therefore l\,(\mathrm{PQ})^2 = (10)^2 - (8)^2$

$\therefore l\,(\mathrm{PQ})^2 = 100 - 64$

$\therefore l\,(\mathrm{PQ})^2 = 36$

$\therefore l\,(\mathrm{PQ})^2 = (6)^2$

$\therefore l\,(\mathrm{PQ}) = 6\ \mathrm{cm}$ …. (I)

Also seg PS ⊥ chord NO

The perpendicular drawn from the centre of a circle to its chord bisects the chord.

$\therefore l\,(\mathrm{NS}) = \frac{1}{2}\,l\,(\mathrm{NO})$

$\therefore l\,(\mathrm{NS}) = \frac{1}{2} \times 12$

$\therefore \boldsymbol{l}\ (\mathbf{NS}) = \mathbf{6\ cm}$

Here, the radius of a circle $= \boldsymbol{l}\ (\mathbf{PN}) = \mathbf{10\ cm}$

Δ **PSN is a right angled triangle.**

$\therefore$ **In** Δ **PSN,**

By Pythagoras theorem,

$\boldsymbol{l}\ (\mathbf{PN})^2 = \boldsymbol{l}\ (\mathbf{NS})^2 + \boldsymbol{l}\ (\mathbf{PS})^2$

$\therefore (\mathbf{10})^2 = (\mathbf{6})^2 + \boldsymbol{l}\ (\mathbf{PS})^2$

$\therefore \boldsymbol{l}\ (\mathbf{PS})^2 = (\mathbf{10})^2 - (\mathbf{6})^2$

$\therefore \boldsymbol{l}\ (\mathbf{PS})^2 = \mathbf{100 - 36}$

$\therefore \boldsymbol{l}\ (\mathbf{PS})^2 = \mathbf{64}$

$\therefore \boldsymbol{l}\ (\mathbf{PS})^2 = (\mathbf{8})^2$

$\therefore \boldsymbol{l}\ (\mathbf{PS}) = \mathbf{8\ cm}$ **….. (II)**

Now, $\boldsymbol{l}\ (\mathbf{PS}) = \boldsymbol{l}\ (\mathbf{PQ}) + \boldsymbol{l}\ (\mathbf{QS})$

$\therefore \mathbf{8 = 6} + \boldsymbol{l}\ (\mathbf{QS})$ **…… [form (I) and (II)]**

$\therefore \boldsymbol{l}\ (\mathbf{QS}) = \mathbf{8 - 6}$

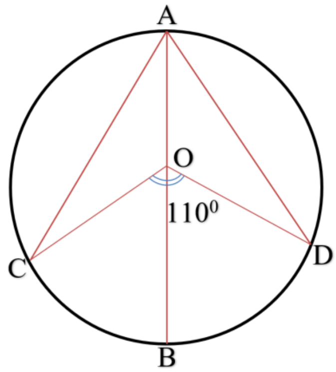
$\therefore \boldsymbol{l}\ (\mathbf{QS}) = \mathbf{2}$

$\therefore \boldsymbol{l}\ (\mathbf{QS})$ **is 2 cm.**

10. In the adjacent figure, O is the centre of the circle. Seg AB is a diameter.

$\mathbf{m\angle COD = 110^0,\ m\angle COB = m\angle BOD}$**, then,**

(i) Find the measure of $\angle$ AOD and $\angle$ AOC



(ii) Show that arc AC $\cong$ arc AD

(iii) Show that chord AC $\cong$ chord AD.

Solution : 'O' is the centre of the circle.

Seg AB is a diameter.

$\mathbf{m\angle COD = 110^0}$ **…… (Given)**

$\mathbf{m\angle COD = m\angle COB + m\angle BOD}$

But, $\mathbf{m\angle COB = m\angle BOD}$

$\therefore \mathbf{m\angle COD = m\angle BOD + m\angle BOD}$

$\therefore \mathbf{110^0 = 2\ m\angle BOD}$

$$\therefore \mathbf{m\angle BOD = \frac{110}{2}}$$

$\therefore \mathbf{m\angle BOD = 55^0}$

$\angle$ AOB is a semicircle of a circle.

Measure of a semicircular arc is 180^0.

$\therefore \mathbf{m\angle AOB = 180^0}$

(i) $\mathbf{m\angle AOD = m\angle AOB - m\angle BOD}$

$\therefore \mathbf{m\angle AOD = 180^0 - 55^0}$

$\therefore \mathbf{m\angle AOD = 125^0}$ **……….(I)**

$\mathbf{m\angle AOC = m\angle AOB - m\angle COB}$

$\therefore \mathbf{m\angle AOC = m\angle AOB - m\angle BOD}$

…..($\because$ $\mathbf{m\angle COB} = \boldsymbol{m}\angle \mathbf{BOD}$**)**

$\therefore \mathbf{m\angle AOC = 180 - 55}$

$\therefore \mathbf{m\angle AOC = 125}$ **……. (II)**

(ii) $\mathbf{m\,(arc\ AC) = m\angle AOC = 125^0}$**…….[From (II)]**

$\mathbf{m\,(arc\ AD) = m\angle AOD = 125^0}$**…….[From (I)]**

$\therefore$ **m (arc AC) ≅ m (arc AD) …….[From (I) & (II)]**

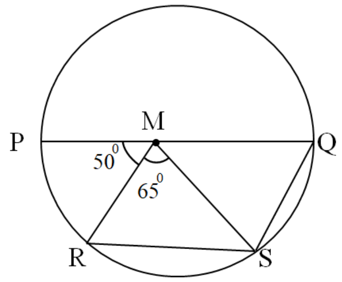
arc AC ≅ arc AD

…..($\because$ **Arcs are of same measures) …(III)**

(iii)arc AC ≅ arc AD…….[From (III)]

chord AC ≅ chord AD…..(Corresponding chords of congruent arcs are congruent.)

11. In the adjacent figure M is the centre of the circle, whose diameter is seg PQ. Measures of some central angles are given in the figure. Hence show that chord RS ≅ chord QS.



Solution : M is the centre of the circle whose diameter is seg PQ.

$m\angle PMR = 50$ and $m\angle RMS = 65$...... (Given)

$\angle PMQ$ is the semicircle.

Measure of a semicircle is 180^0

$\therefore m\angle PMQ = 180^0$

$$m\angle SMQ = m\angle PMQ - m\angle PMR - m\angle RMS$$

$$= 180^0 - 50^0 - 65^0$$

$$= 180^0 - 115^0$$

$\therefore m\angle SMQ = 65^0$

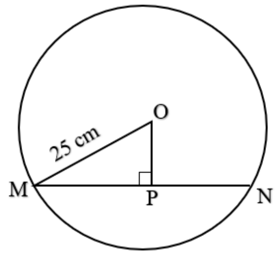
$\therefore m\angle RMS = m\angle SMQ = 65^0$

$\therefore m(arc\ RS) = m\ (arc\ QS)$

$\therefore arc\ RS \cong arc\ QS$ ..... (Arcs are of same measures)

∴ Chord RS ≅ Chord QS ….. (Corresponding chords of congruent arcs are congruent)

12. In the alongside figure, O is the centre of the circle. The length of the chord is 40 cm. The point P is the midpoint of that chord. Find *l* (OP)



Solution : The radius of a circle $= l\,(\mathrm{OM}) = 25$ cm

….. (Given)

O is the centre of the circle. MN is the chord.

$l\,(\mathrm{MN}) = 40$ cm

P is the midpoint of chord MN

∴ Seg OP ⊥ chord MN

$\therefore l\,(\mathrm{MP}) = \frac{1}{2} \times l\,(\mathrm{MN})$

$\therefore l\,(\mathrm{MP}) = \frac{1}{2} \times 40$

$\therefore l\,(\mathrm{MP}) = 20$ cm

Δ OMP is a right angled triangle.

In Δ OMP,

By Pythagoras theorem,

$l\,(\mathrm{OM})^2 = l\,(\mathrm{OP})^2 + l\,(\mathrm{MP})^2$

$\therefore (25)^2 = l\,(\mathrm{OP})^2 + (20)^2$

$\therefore l\,(\mathrm{OP})^2 = (25)^2 - (20)^2$

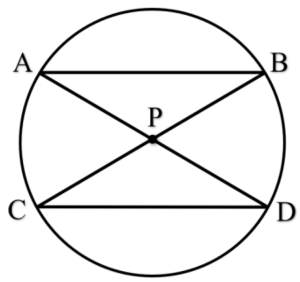
$\therefore l\,(\mathrm{OP})^2 = 625 - 400$

$\therefore\ l\,(\mathrm{OP})^2 = 225$

$\therefore l\,(\mathrm{OP})^2 = (15)^2$

$\therefore l\,(\mathrm{OP})$ is 15 cm.

13. In the adjacent figure, with centre P, $\angle \mathrm{APB} = 120^0$. Then find m $\angle$ CPD and m $\angle$ BCD.



Solution : m $\angle \mathrm{APB} = 120^0$ …… (Given)

$\angle \mathrm{APB} \cong \angle \mathrm{CPD}$ ….. (Opposite angles)

$\therefore$ m $\angle \mathrm{APB} =$ m $\angle \mathrm{CPD} = 120^0$

Seg PC $\cong$ Seg PD …… (Radius of same circle)

$\therefore$ m $\angle \mathrm{PCD} =$ m $\angle \mathrm{PDC}$ ….. (Angle of congruent opposite side)

In Δ CPD,

m $\angle$ CPD + m $\angle$ PCD + m $\angle$ PDC $= 180^0$

….(The sum of the three angles of a triangle is 180^0.)

$\therefore 120 + m\angle PCD + m\angle PDC = 180^0$

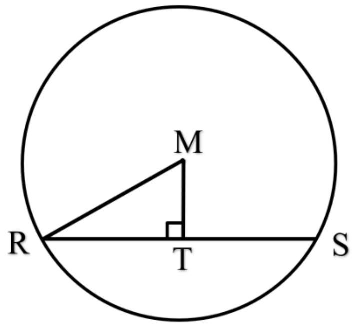
$$\therefore 2\ m\angle PDC = 180 - 120$$

$$\therefore 2\ m\angle PDC = 60$$

$$\therefore m\angle PDC = \frac{60}{2}$$

$$\therefore m\angle PDC = 30$$

14. Radius of a circle with centre M is 26 cm. Find the length of the chord, if the chord is at a distance of 10 cm from the centre of the circle.



Solution : The distance of the chord from the centre means the length of the segment which is perpendicular drawn from the centre of the circle. 'O' is the centre of the circle whose chord is seg RS. Seg MT ⊥ chord RS

Radius of a circle $= l\,(MR) = 26$ cm, $l\,(MT) = 10$ cm

Δ MTR is a right angled triangle.

In Δ MTR,

By Pythagoras theorem,

$$l\,(MR)^2 = l\,(MT)^2 + l\,(RT)^2$$

$\therefore (26)^2 = (10)^2 + l\,(RT)^2$

$\therefore l\,(RT)^2 = (26)^2 - (10)^2$

$\therefore l\,(RT)^2 = 676 - 100$

$\therefore l\,(RT)^2 = 576$

$\therefore l\,(RT)^2 = (24)^2$

$\therefore l\,(RT) = 24 \text{ cm}$

The perpendicular drawn from the centre of a circle to its chord bisects the chord.

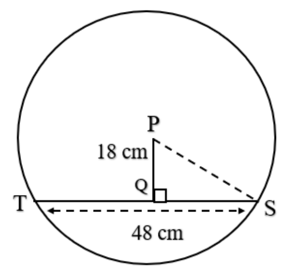
$\therefore l\,(RS) = 2 \times l\,(RT)$

$= 2 \times 24$

$= 48$

∴ The length of the chord is 48 cm.

15. P is the centre of the circle. The chord of a circle 48 cm. It is distance from the centre is 18 cm. Find the radius of the circle.



Solution : Join Seg PS.

$l\,(PQ) = 18$ cm …… (Given)

TS is chord of a circle with centre P.

$\therefore l\,(\mathrm{TS}) = 48$ cm .... (Given)

Point Q is the midpoint of the chord

Seg PQ $\perp$ Chord TS

The perpendicular drawn from the centre of a circle to its chord bisects the chord.

$$\therefore l\,(\mathrm{QS}) = \frac{1}{2} \times l\,(\mathrm{TS})$$

$$\therefore l\,(\mathrm{QS}) = \frac{1}{2} \times 48$$

$$\therefore l\,(\mathrm{QS}) = 24 \text{ cm}$$

Δ PQS is a right angled triangle

In right angled Δ PQS,

By Pythagoras theorem,

$$l\,(\mathrm{PS})^2 = l\,(\mathrm{PQ})^2 + l\,(\mathrm{QS})^2$$

$$\therefore l\,(\mathrm{PS})^2 = (18)^2 + (24)^2$$

$$= 324 + 576$$

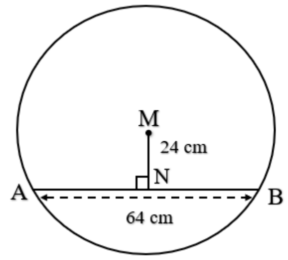
$$= 900$$

$$\therefore l\,(\mathrm{PS})^2 = (30)^2$$

$$\therefore l\,(\mathrm{PS}) = 30 \text{ cm}$$

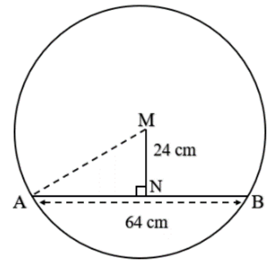
∴ The radius of a circle is 30 cm.

16. ‘M’ is the centre of the circle whose chord is 64 cm long. Its distance from the centre is 24 cm. Find the length of the longest chord of the circle.



Solution : Join seg MA

l (AB) = 64 cm, l (MN) = 24 cm



M is the centre of the circle whose chord is AB.

Point N is the midpoint of the chord.

Seg MN ⊥ Chord AB

The perpendicular drawn from the centre of the circle to the chord bisects the chord.

$$\therefore l\,(AN) = \frac{1}{2}\, l\,(AB)$$

$$\therefore l\,(AN) = \frac{1}{2} \times 64$$

$$\therefore l\,(AN) = 32 \text{ cm}$$

Δ MNA is a right angled triangle.

∴ In Δ MNA ,

By Pythagoras theorem,

$$\boldsymbol{l}\ (\mathrm{MA})^2 = \boldsymbol{l}\ (\mathrm{MN})^2 + \boldsymbol{l}\ (\mathrm{AN})^2$$

$$\therefore \boldsymbol{l}\ (\mathrm{MA})^2 = (24)^2 + (32)^2$$

$$= 576 + 1024$$

$$= 1600$$

$$\therefore \boldsymbol{l}\ (\mathrm{MA})^2 = (40)^2$$

$$\therefore \boldsymbol{l}\ (\mathrm{MA}) = 40 \text{ cm}$$

A diameter of the circle is the longest chord of the circle.

$$\therefore \text{Diameter of the circle } = 2 \times \text{radius}$$

$$= 2 \times \boldsymbol{l}\ (\mathrm{MA})$$

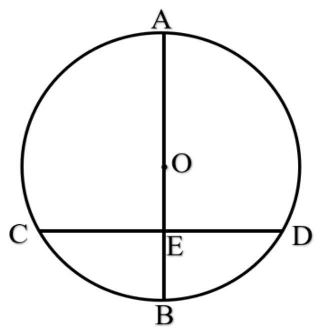
$$= 2 \times 40$$

$$= 80 \text{ cm}$$

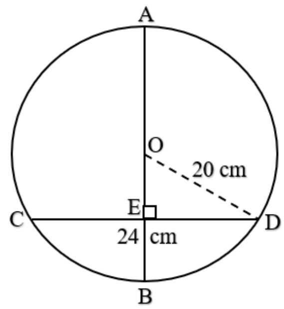
$\therefore$ The length of the longest chord of the circle is 80 cm.

17. ‘O’ is the centre of the circle.

Seg AB is diameter and Seg CD is the chord. Seg AB and Seg CD are intersect at point E. If $\boldsymbol{l}\ (\mathrm{CD}) = 24$ cm and radius is 20 cm then find the length of AE.



Solution : Join OD.



$l\,(\mathrm{OD}) = 20$ cm, $l\,(\mathrm{CD}) = 24$ cm

The point E is the midpoint of the diameter AB and chord CD.

The perpendicular drawn from the centre of the circle to the chord bisects the chord.

$\therefore l\,(\mathrm{ED}) = \frac{1}{2}\, l\,(\mathrm{CD})$

$= \frac{1}{2} \times 24$

$\therefore l\,(\mathrm{ED}) = 12$ cm

Δ OED is a right angled triangle.

$\therefore$ In a right angled Δ OED,

By Pythagoras theorem,

$l\,(\mathrm{OD})^2 = l\,(\mathrm{OE})^2 + l\,(\mathrm{ED})^2$

$\therefore (20)^2 = l\,(\mathrm{OE})^2 + (12)^2$

$\therefore l\,(\mathrm{OE})^2 = (20)^2 - (12)^2$

$= 400 - 144$

$\therefore l\,(\mathrm{OE})^2 = 256$

$\therefore l\,(\mathrm{OE}) = 16$ cm

Now, $l\,(\mathrm{AE}) = l\,(\mathrm{AO}) + l\,(\mathrm{OE})$

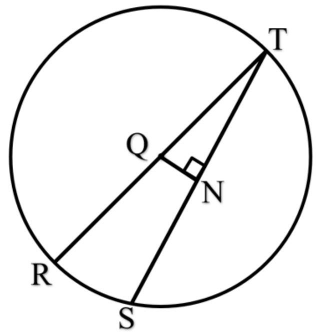
But, l (AO) = l (OD) = Radius of a circle.

$\therefore l\,(AE) = 20 + 16$

$\therefore l\,(AE) = 36$ cm

∴ The length of AE is 36 cm.

18. In the adjoining figure, Seg RT is a diameter and Seg ST is a chord of a circle with centre 'Q'. If l (QN) = 25cm , l (ST) = 120 cm then find the length of Seg RT.



Solution : Seg RT is a diameter and Seg ST is a chord of a circle with centre 'Q'

$l\,(QN) = 25$ cm, $l\,(ST) = 120$ cm …… (given)

Seg QN ⊥ Seg ST

The perpendicular drawn from the centre of the circle to the chord bisects the chord.

$$\therefore l\,(NT) = \frac{1}{2}\, l\,(ST)$$

$$\therefore l\,(NT) = \frac{1}{2} \times 120$$

$\therefore\ l\ (\mathrm{NT}) = 60\ \mathrm{cm}$

Now, Δ QNT is a right angled triangle

∴ In Δ QNT,

By Pythagoras theorem,

$$l\ (\mathrm{QT})^2 = l\ (\mathrm{QN})^2 + l\ (\mathrm{NT})^2$$

$$\therefore\ l\ (\mathrm{QT})^2 = (25)^2 + (60)^2$$

$$= 625 + 3600$$

$$\therefore\ l\ (\mathrm{QT})^2 = 4225$$

$$\therefore\ l\ (\mathrm{QT})^2 = (65)^2$$

$$\therefore\ l\ (\mathrm{QT}) = 65\ \mathrm{cm}$$

∴ Radius of a circle $= l\ (\mathrm{QT}) = 65\ \mathrm{cm}$

Seg RT is a diameter of the circle.

∴ Diameter $= 2 \times$ **Radius**

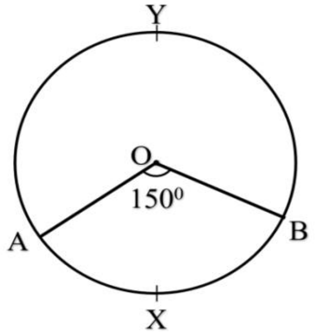
$$\therefore\ l\ (\mathrm{RT}) = 2 \times l\ (\mathrm{QT})$$

$$\therefore\ l\ (\mathrm{RT}) = 2 \times 65$$

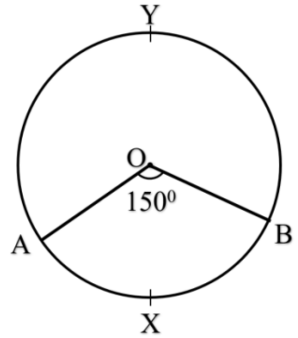
$$\therefore\ l\ (\mathrm{RT}) = 130\ \mathrm{cm}$$

∴ The length of the seg RT is 130 cm.

19. In the figure, O is the centre of the circle. If the measure of a central angle is 150^0, find the measure of corresponding minor arc and that of the major arc corresponding to this arc.



Solution : The measure of a central angle is 150^0 with centre O of the circle.



$\therefore$ m $\angle$ AOB $= 150^0$

In the figure,

The minor arc AXB corresponding to the central angle.

$\therefore$ m (arc AXB) = m $\angle$ AOB

$\therefore$ m (arc AXB) $= 150^0$ …. (I)

Now, arc AYB is the major arc corresponding to the central angle AOB.

Measure of a major arc = 360^0 − Measure of corresponding minor arc

m (arc AYB) $= 360^0$ − m (arc AXB)

…… (Arc AXB and arc AYB are the corresponding arcs)

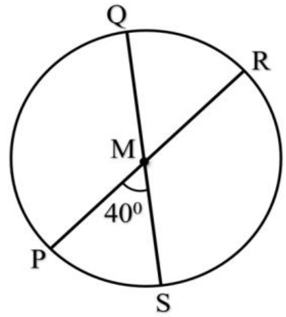
m (arc AYB) $= 360^0 - 150^0$ …… [from (I)]

$= 210^0$

∴ The measure of corresponding minor arc is 150^0 and the measure of the corresponding major arc is 210^0.

20. 'M' is the centre of the circle.

$m\angle AOP = 40^0$ Then,



(i) Find the measure of arc PS and arc QR

(ii) Are the arc PS and arc QR congruent ? Give reason

(iii) Are the arc PQ and arc SR congruent ? Give reason.

Solution :'M' is the centre of the circle. Diameters PR and SQ intersect at point M.

$m\angle PMS = 40^0$

(i) $m\,(arc\ PS) = m\angle PMS = 40^0$

……. (Corresponding central angle)

$\angle PMS$ and $\angle QMR$ are the opposite angles.

$\therefore m\angle PMS = m\angle QMR = 40^0$

$\therefore m\angle QMR = 40^0$

Now, m (arc QR) $= m\angle QMR$

….. (Corresponding central angle)

$= 40^0$

(ii) $m\angle PMS = 40^0$, $m\angle QMR = 40^0$

$\therefore$ m (arc PS) = m (arc QR)

$\therefore$ (arc PS) ≅ m (arc QR) …(Arcs of the same measures)

Reason: **Arc PS and arc QR are congruent because the measure of the central angle corresponding to the arcs are same that means each are 40^0.**

(iii) Seg SQ is a diameter.

$\therefore \mathbf{m\angle PMS + m\angle QMP = 180^0}$

….. (Angles in a linear pair)

$\therefore \mathbf{40^0 + m\angle QMP = 180^0}$ **…. ($\because$ $\mathbf{m\angle PMS = 40^0}$)**

$\therefore \mathbf{m\angle QMP = 180^0 - 40^0}$

$\therefore \mathbf{m\angle QMP = 140^0}$

Also, Seg PR is a diameter.

$\therefore \mathbf{m\angle PMS + m\angle RMS = 180^0}$

… (Angles in a linear pair)

$\therefore \mathbf{40^0 + m\angle RMS = 180^0}$

$\therefore \mathbf{m\angle RMS = 180^0 - 40^0}$

$\therefore \mathbf{m\angle RMS = 40^0}$

M (arc PQ) = m ∠ QMP $= 140^0$

….. (Corresponding central angle)

M (arc SR) = m ∠ RMS $= 140^0$

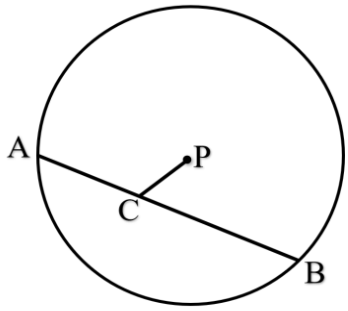
…… (Corresponding central angle)

∴ m (arc PQ) = m (arc SR)

∴ Arc PQ ≅ Arc SR ….. (Arcs are of same measures)

Reason: Arc PQ and arc SR are congruent because the measure of the central angle corresponding to the arcs are same i.e. each are 140^0.

21. In the figure alongside, seg AB is a chord of a circle with centre P.



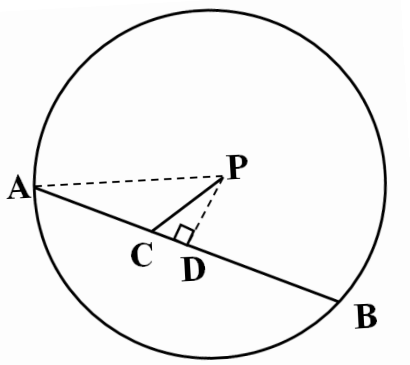
l **(AB) = 25 cm,** ***l*** **(AC) = 9 cm**

l **(PC) = 5 cm then find the radius of the circle. Complete the following activity.**

Activity :

Join PD ⊥ AB

Join PA.



Seg AB is a ……. of a circle with centre P.

$l\ (AB) = \square$, $l\ (AC) = \square$, $l\ (PC) = \square$

D is the midpoint of chord AB.

$\therefore l\ (AD) = \frac{1}{2} \times \square$ …… **(Formula)**

$= \frac{1}{2} \times \square$ …… **(By putting value)**

$\therefore l\ (AD) = \square$ **cm.**

But $l\ (CD) = l\ (AD) - l\ (AC)$

$\therefore l\ (CD) = \square - \square$ …. **(By putting values)**

$\therefore l\ (CD) = \frac{\square - \square}{2}$

$\therefore l\ (CD) = \frac{\square}{2}$ **cm**

Δ **PDC is a right angled triangle**

$\therefore$ **In** Δ **PDC,**

By Pythagoras theorem,

$l\ (PC)^2 = \square + \square$

$\square = \square + \left(\frac{7}{2}\right)^2$ ……. **(By putting values)**

$\therefore l\ (PD)^2 = \square - \left(\frac{7}{2}\right)^2$

$$= \square - \frac{49}{4}$$

$$= \frac{\square - 49}{4}$$

$$\therefore l\,(PD)^2 = \frac{\square}{4}$$

$$l\,(PD) = \frac{\square}{2}\ \text{cm}$$

Now Δ PDA is a right angled triangle.

∴ In Δ PDA ,

By Pythagoras theorem,

$$\therefore \square = l\,(PD)^2 + \square$$

$$= \left(\frac{\square}{2}\right)^2 + (\square)^2 \quad \text{...... (By putting values)}$$

$$= \frac{\square}{4} + \square$$

$$= \frac{\square + \square}{4}$$

$$= \frac{\square}{4}$$

$$= \square$$

$$\therefore l\,(PA)^2 = (13)^2$$

$$\therefore l\,(PA) = \square\ \text{cm}$$

$\therefore$ **Radius of a circle is ……..**

Solution: Join PD $\perp$ AB

Join PA.

Seg AB is a <u>Chord</u> of a circle with centre P.

$l\ (AB) = \boxed{25\ cm}$, $l\ (AC) = \boxed{9\ cm}$, $l\ (PC) = \boxed{5\ cm}$

D is the midpoint of chord AB.

$\therefore l\ (AD) = \frac{1}{2} \times \boxed{l\ (AB)}$ **…… (Formula)**

$= \frac{1}{2} \times \boxed{25}$ **…… (By putting value)**

$\therefore l\ (AD) = \boxed{\frac{25}{2}}$ cm.

But $l\ (CD) = l\ (AD) - l\ (AC)$

$\therefore l\ (CD) = \boxed{\frac{25}{2}} - \boxed{9}$ **…. (By putting values)**

$\therefore l\ (CD) = \frac{\boxed{25} - \boxed{18}}{2}$

$\therefore l\ (CD) = \frac{\boxed{7}}{2}$ cm

Δ PDC is a right angled triangle.

$\therefore$ **In Δ PDC ,**

By Pythagoras theorem,

$$l\,(PC)^2 = \boxed{l\,(PD)^2} + \boxed{l\,(CD)^2}$$

$$\boxed{(5)^2} = \boxed{l\,(PD)^2} + \left(\frac{7}{2}\right)^2$$ .... **(By putting values)**

$$\therefore l\,(PD)^2 = \boxed{(5)^2} - \left(\frac{7}{2}\right)^2$$

$$= \boxed{25} - \frac{49}{4}$$

$$= \frac{\boxed{100} - 49}{4}$$

$$\therefore l\,(PD)^2 = \frac{\boxed{51}}{4} \text{ cm.}$$

$$l\,(PD) = \frac{\boxed{\sqrt{51}}}{2} \text{ cm.}$$

Now Δ PDA is a right angled triangle .

∴ In Δ PDA ,

By Pythagoras theorem,

$$\therefore \boxed{l\,(PA)^2} = l\,(PD)^2 + \boxed{l\,(AD)^2}$$

$$= \boxed{\left(\frac{\sqrt{51}}{2}\right)^2} + \boxed{\left(\frac{25}{2}\right)}^2$$ ...... **(By putting values)**

$$= \frac{\boxed{51}}{4} + \boxed{\frac{625}{4}}$$

$$= \frac{\boxed{51} + \boxed{625}}{4}$$

$= \frac{\boxed{676}}{4}$

$= \boxed{169}$

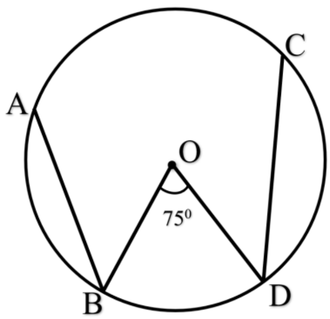
$\therefore l(PA)^2 = (13)^2$

$\therefore l(PA) = \boxed{13}$ cm

$\therefore$ **The radius of a circle is 13 cm.**

22. In the adjoining figure, O is the centre of the circle. Chord AB $\cong$ Chord CD.

If m $\angle$ BOD = 75^0 and m (arc AB) = 85^0 then complete the following activity to find the measures of



(i) arc BD (ii) arc AC (iii) arc ACD

Activity :

m $\angle$ BOD = $\boxed{}$, m (arc AB) = $\boxed{}$

Chord $\boxed{}$ $\cong$ Chord $\boxed{}$ of a circle with centre O .

$\therefore$ **arc $\boxed{}$ $\cong$ arc $\boxed{}$ … (Congruent chords of a circle subtend congruent angles.)**

$\therefore$ **m (arc $\boxed{}$) = m (arc $\boxed{}$)**

$\therefore$ **m (arc $\boxed{}$) = m (arc $\boxed{}$) = 85^0**

(i) m (arc BD) = m ($\angle \boxed{}$) = $\boxed{}$

.... (Corresponding central angle)

(ii) Measure of a circle is 360^0.

m (arc BD) + m (arc AB) + m (arc AC) + m (arc CD) = $\boxed{}$

$\therefore \boxed{} + 85 + m\ (\text{arc } AC) + \boxed{} = \boxed{}$

$\therefore \boxed{} + m\ (\text{arc } AC) = \boxed{}$

$\therefore\ m\ (\text{arc } AC) = \boxed{} - \boxed{} = \boxed{}$

(iii) m (arc ACD) = m (arc $\boxed{}$) + m (arc $\boxed{}$)

$= \boxed{} + \boxed{}$

$\therefore$ m (arc ACD) = $\boxed{}$

Solution :

$m \angle BOD = \boxed{75^0}$, m (arc AB) = $\boxed{85^0}$

Chord $\boxed{AB} \cong$ Chord $\boxed{CD}$ of a circle with centre O .

$\therefore$ arc $\boxed{AB} \cong$ arc $\boxed{CD}$ ... (Congruent chords of a circle subtend congruent angles.)

$\therefore m\ (\text{arc } \boxed{AB}) = m\ (\text{arc } \boxed{CD})$

$\therefore m\ (\text{arc } \boxed{AB}) = m\ (\text{arc } \boxed{CD}) = 85^0$

(i) m (arc BD) = m (∠ $\boxed{\text{BOD}}$) = $\boxed{75^0}$

.... (Corresponding central angle)

(ii) Measure of a circle is 360^0.

m (arc BD) + m (arc AB) + m (arc AC) + m (arc CD) = $\boxed{360^0}$

$\therefore \ \boxed{75^0} + 85^0 + \text{m (arc AC)} + \boxed{85^0} = \boxed{360^0}$

$\therefore \ \boxed{245^0} + \text{m (arc AC)} = \boxed{360^0}$

$\therefore \ \text{m (arc AC)} = \boxed{360^0} - \boxed{245^0} = \boxed{115^0}$

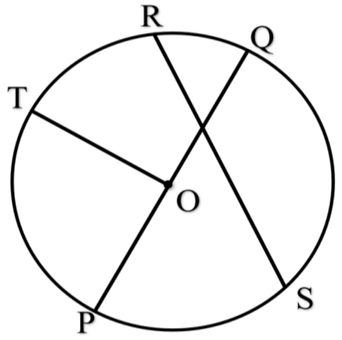
(iii) m (arc ACD) = m (arc $\boxed{\text{AC}}$) + m (arc $\boxed{\text{CD}}$)

$= \boxed{115^0} + \boxed{85^0}$

$\therefore$ m (arc ACD) = $\boxed{200^0}$

23. Fill in the blank :

O is the centre of the circle.



(1) Seg OT is ...... of the circle.

(2) Seg PQ is the .... of the circle.

(3) Seg RS is the .... of the circle.

(4) The length of seg PQ is ...... than the length of seg OT

Ans :

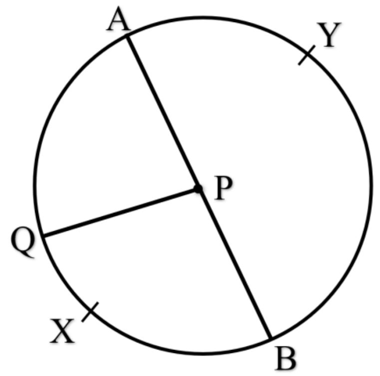
(1) Seg OT is <u>radius</u> of the circle.

(2) Seg PQ is the <u>diameter</u> of the circle.

(3) Seg RS is the <u>chord</u> of the circle.

(4) The length of seg PQ is <u>doubled</u> than the length of seg OT.

24. In the adjacent figure point P the centre of the circle. Observe the diagram and fill in the blanks.



(1) ……. is the central angle.

(2) Name of minor arcs : ……

(3) Name of major arcs : ……

(4) Semicircle : ……

(5) m (arc AQ) = m ∠ ……

(6) m (arc ABQ) = $360^0 -$ m ∠ ……

Ans : (1) <u>∠ APQ</u> is the central angle.

(2) Names of minor arcs : <u>Arc QXB , Arc AQ</u>

(3) Names of major arcs : <u>Arc QAB, Arc ABQ</u>

(4) Semicircle : <u>Arc AYB, Arc AXB</u>

(5) m (arc AQ) = m ∠ <u>APQ</u>

(6) m (arc ABQ) = 360 – m ∠ APQ

25. Write the following statements are true or false.

1) The right angle drawn from the centre of a circle to its chord bisects the chord.

Ans : False, The perpendicular drawn from the centre of a circle to its chord bisects the chord.

2) The segment joining the centre of a circle and midpoint of its chord is perpendicular to the chord.

Ans : True

3) If the measures of two arcs of circle are same then two arcs are 90^0.

Ans : False, If the measures of two arcs of circle are same then two arcs are congruent.

4) The chords corresponding to congruent arcs are congruent.

Ans : True

5) If two chords are congruent then their corresponding minor arcs and major arcs are not congruent.

Ans : False, If two chords are congruent then their corresponding minor arcs and major arcs are congruent.

6) A diameter is the longest chord of the circle.

Ans : True

7) A circle has limited chords.

Ans : False, A circle has infinite chords.

8) A circle has infinite diameters.

Ans : True.

9) A circle has infinite radii.

Ans : True.

10) A circle divides in two equal parts due to chord which is not a diameter.

Ans : False, A circle divides in two unequal parts due to chord which is not a diameter.

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